5444, [0][1]*

on a constant negative curvature surface

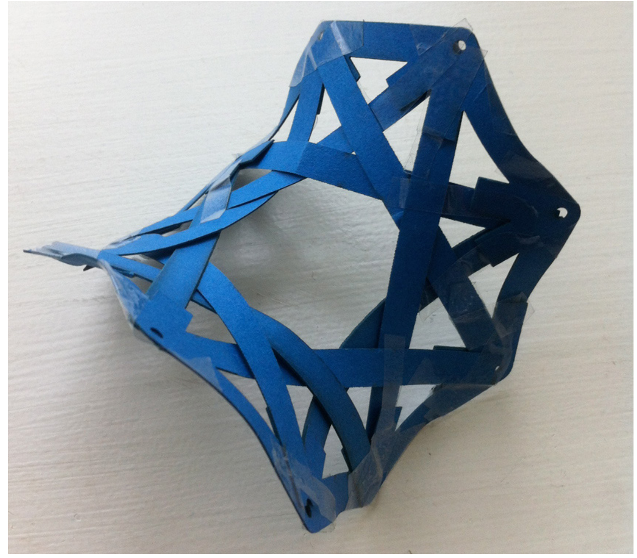
Chaim Goodman-Strauss and Eugene Sargent

G4GB, March 2014

To Sarah Garvin

The intrinsic geometry of this surface is forced by the lengths and angles of its pieces.
Any surface with this instrinsic structure must buckle and bend in this general way.
The paper model above is the basis of the specific extrinsic geometry of the final steel sculpture.

We build surfaces of negative curvature, from strips of flat material: The celebrated Gauss-Bonet Theorem demonstrates that the total curvature of a disk-like region of a surface (for example, one of our units) is precisely captured by measuring the turning excess or deficit around its boundary. For example, consider this decagon with ten 120° angles; as we go around its boundary, we turn 60° ten times, for a total of 600° — an excess of 240° over the customary 360° for flat surfaces. This excess is a precise measurement of the total negative curvature across the decagon.



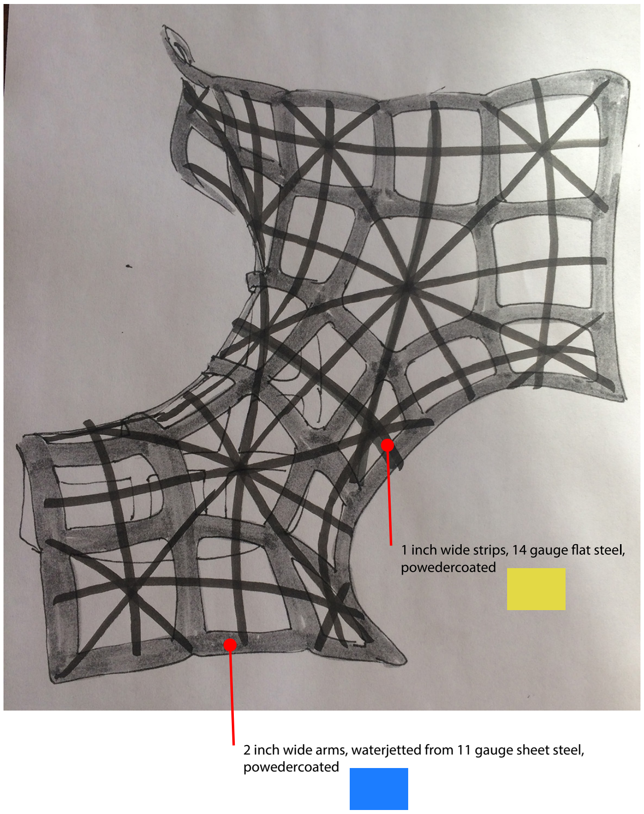
We can control this with exquisite precision. At left is a recent sculpture of a constant negative curvature surface made from pentagons and squares; the angles at the corners are worked out precisely so that the total curvature per unit of area (i.e. the Gaussian curvature) is the same across the entire surface.

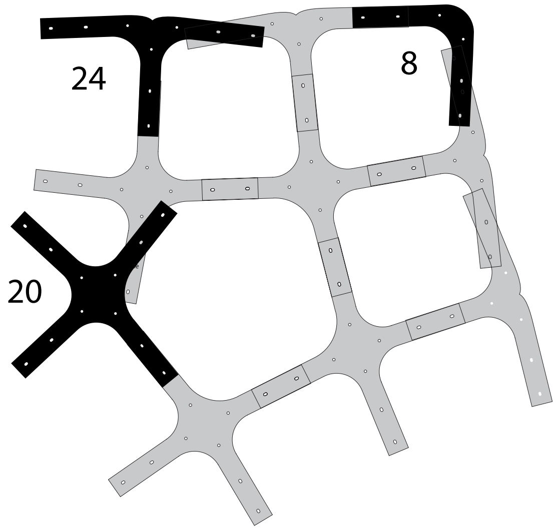
The same piece of tiling is shown in the Poincaré disk below, only differing in scale and placement, but fundamentally, instrinsically, the same geometry.

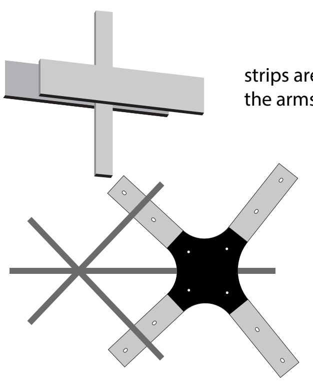
Archimedean of the form
*[0][1] (5,4,4,4)

Equilateral polygons;
the vertex angles are
85.8676° 102.397°

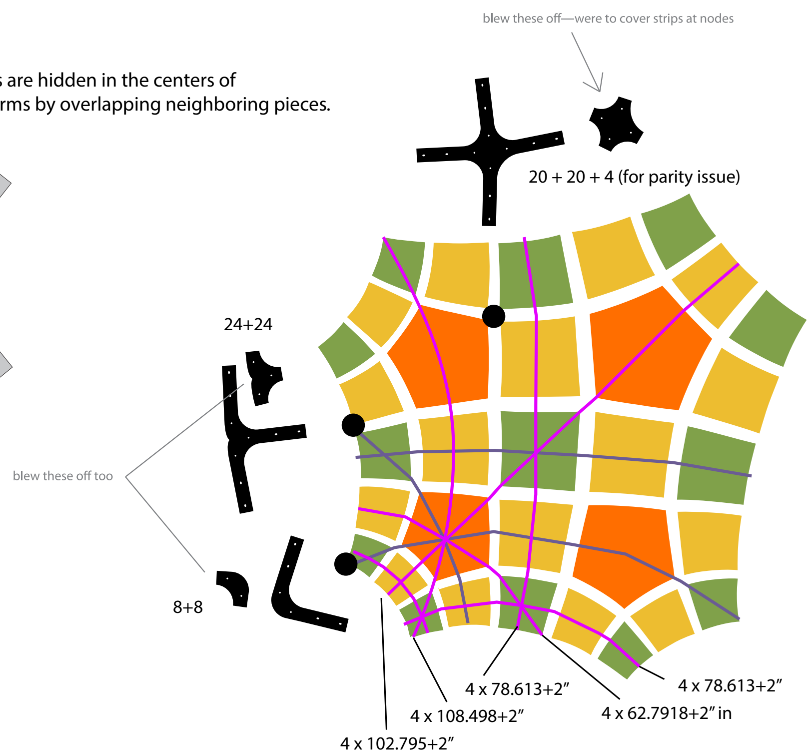
Detail Work ...

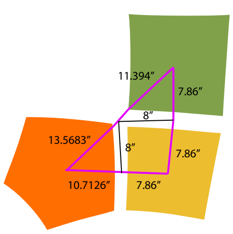






strips are hidden in the centers of the arms by overlapping neighboring pieces.



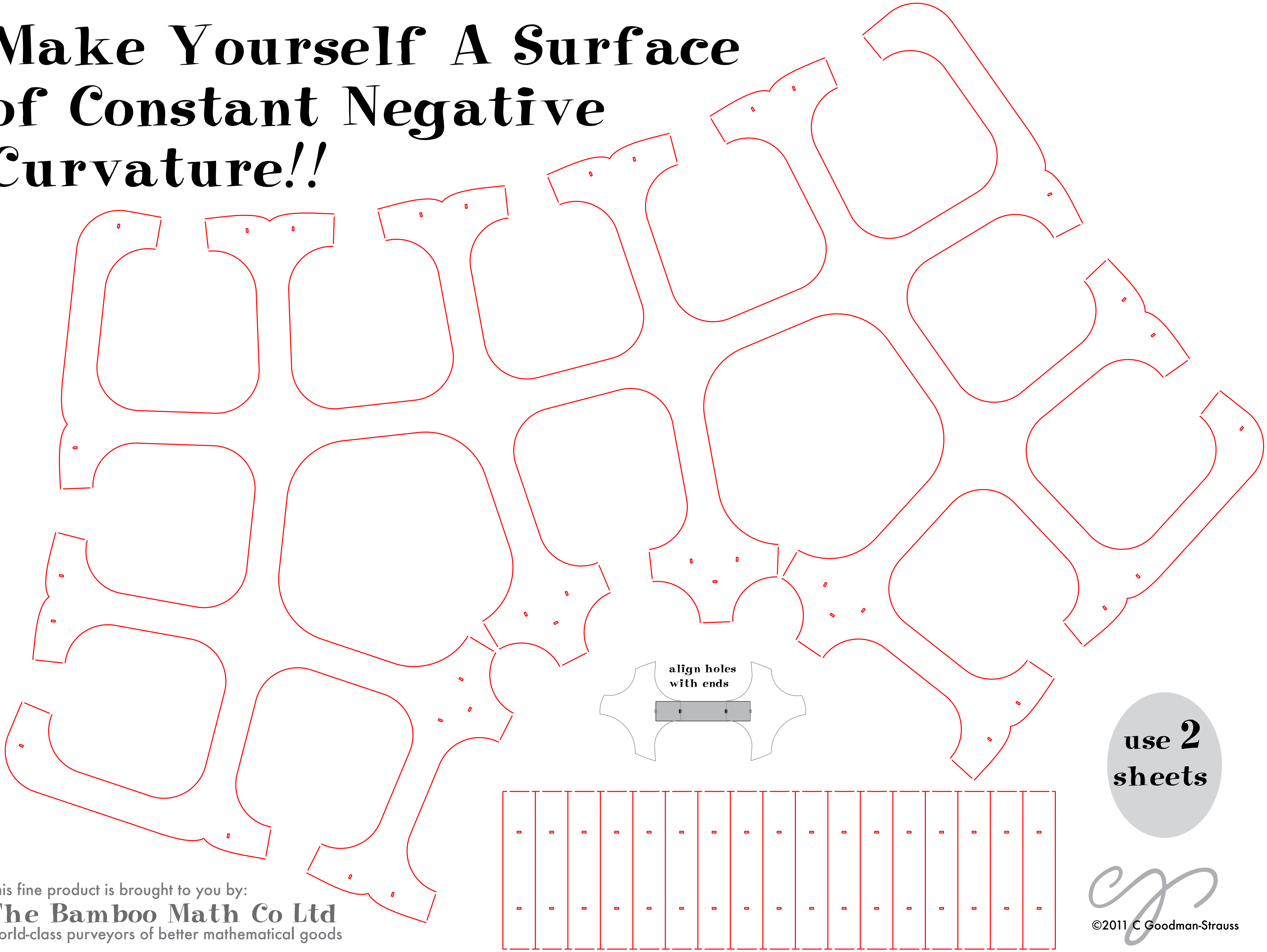


Cut up the sheet opposite,
along the red lines.

Assemble as pictured,

and make yourself a surface of
constant negative curvature!!

Make Yourself A Surface of Constant Negative Curvature!!



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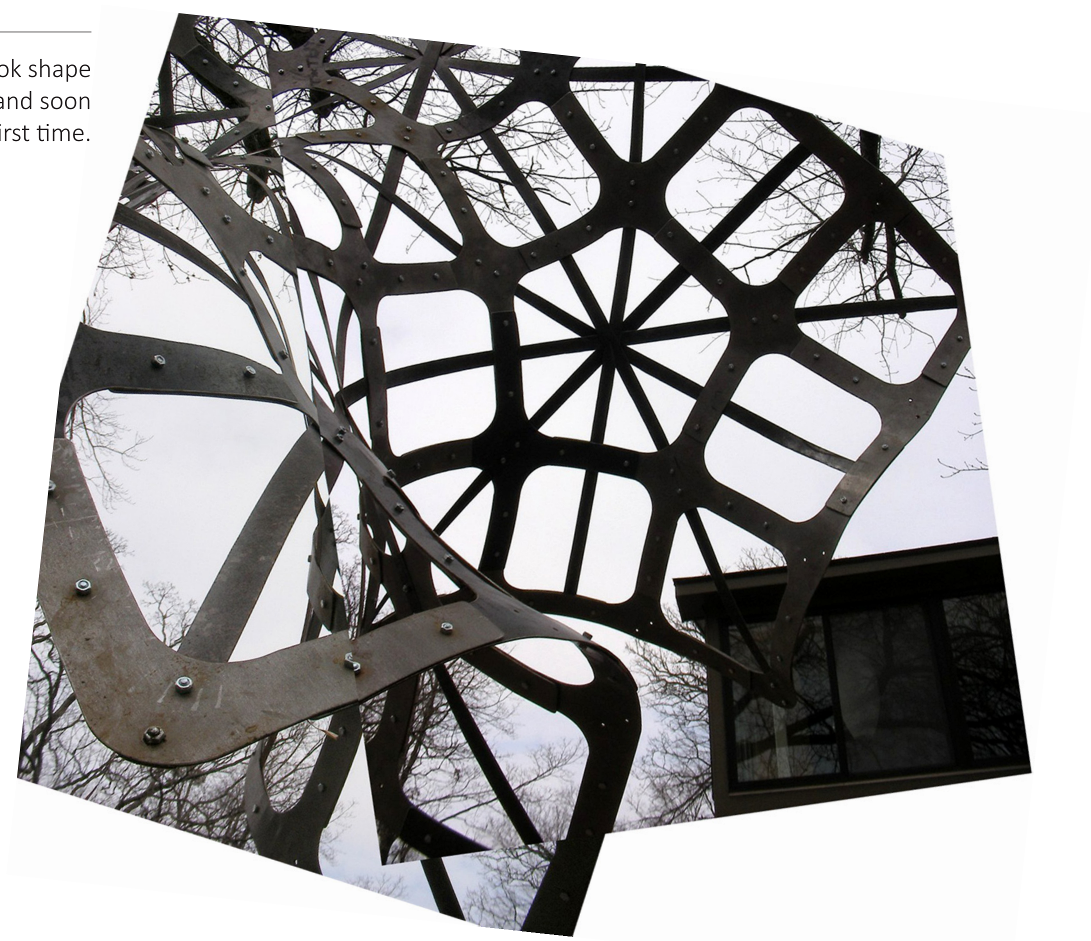
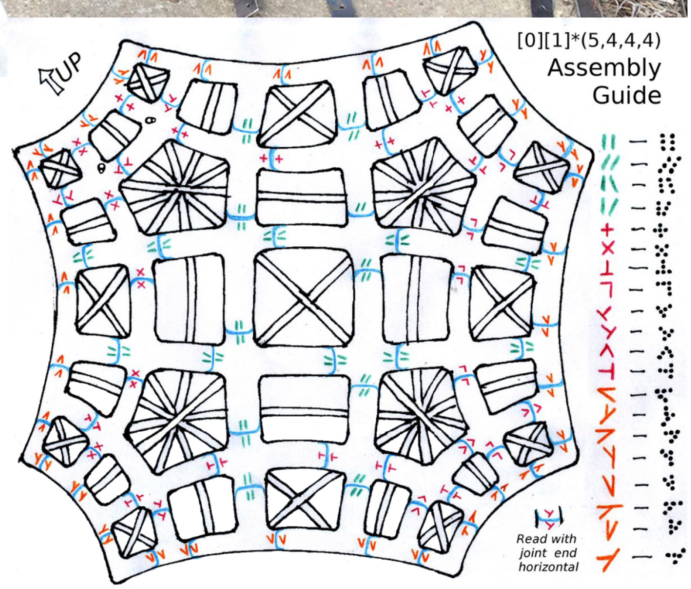
After water-jetting out the pieces, Eugene began to model each one off of a corresponding piece in the paper model.

His specially made bending machine, first used for the Gyring Gyroid, sure came in handy.

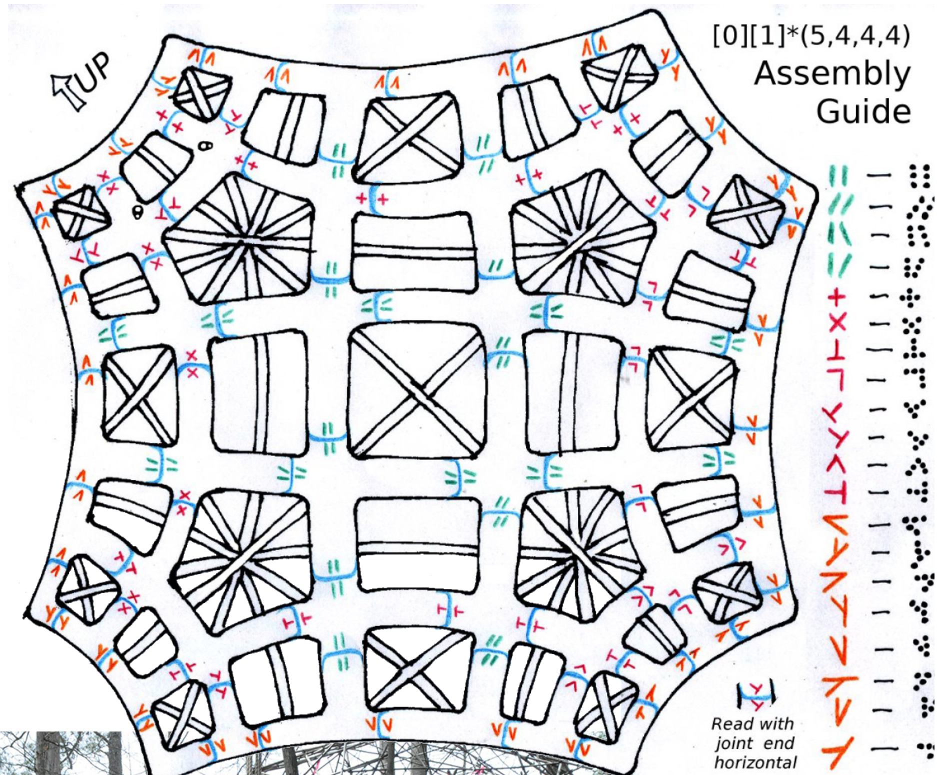
The sculpture took shape in the driveway, and soon was hung for the first time.





... then to be carefully coded, and taken apart.

This simple little crib sheet made everything a snap!



Or maybe not.

This code reads the same upside down as right side up, but not for right-to-left nor up-to-down nor on the diagonal.

It is designed to be easy to drill into dozens of pieces, quickly and to withstand powder coating.

But it made most people cross-eyed, including the sculptors.

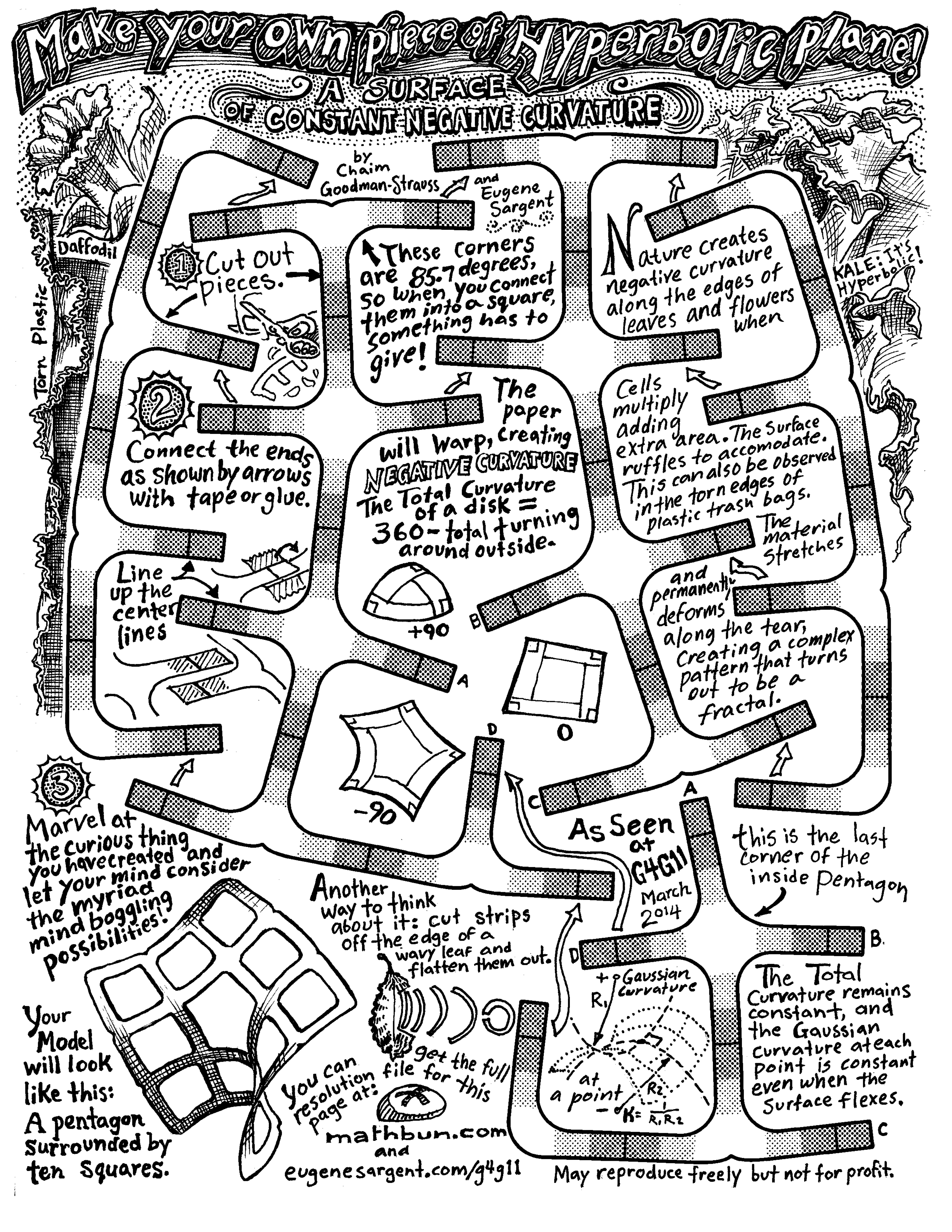
Oh dear.

Heroic efforts saved the day!

And soon it came together...


ROBOTIC
FACTORY

The original paper model (yellow), and the sculpture; the lime green model is at 1/√2 scale to the yellow.


Make your own piece of Hyperbolic plane!
A SURFACE OF CONSTANT NEGATIVE CURVATURE
by Chaim Goodman-Strauss and Eugene Sargent
1 Cut out pieces.
2 Connect the ends as shown by arrows with tape or glue.
Line up the center lines
3 Marvel at the curious thing you have created and let your mind consider the myriad mind boggling possibilities!
These corners are 85.7 degrees, so when you connect them into a square, something has to give!
The paper will warp, creating NEGATIVE CURVATURE
The Total Curvature of a disk = 360 − total turning around outside.
+90
0
−90
Nature creates negative curvature along the edges of leaves and flowers when
Cells multiply adding extra area. The Surface ruffles to accomodate. This can also be observed in the torn edges of plastic trash bags.
The material stretches and permanently deforms along the tear, creating a complex pattern that turns out to be a fractal.
Daffodil
Torn Plastic
KALE: It's Hyperbolic!
A
B
C
D
As Seen at G4G11 March 2014
this is the last corner of the inside pentagon
Another way to think about it: cut strips off the edge of a wavy leaf and flatten them out.
+ Gaussian Curvature
R_1
R_2
at a point
− $K = \frac{1}{R_1 R_2}$
The Total Curvature remains constant, and the Gaussian curvature at each point is constant even when the Surface flexes.
Your Model will look like this: A pentagon surrounded by ten squares.
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