Matching Rules for the Sphinx substitution tiling.

C Goodman-Strauss, August 14, 2003

A tiling by sphinx tiles is a sphinx substitution tiling if and only if every finite patch looks like a patch in some inflated sphinx supertile.

Consider the set of tiles illustrated on the following pages. The matching rules are simply that colors match.

Theorem: the following set of tiles "enforce" the sphinx substitution system.

That is, the tiles

1) do admit a tiling
2) nearly every * such tiling can be "recomposed" into a sphinx substitution tiling.
3) every sphinx tiling can be "recomposed" into a tiling by these tiles.

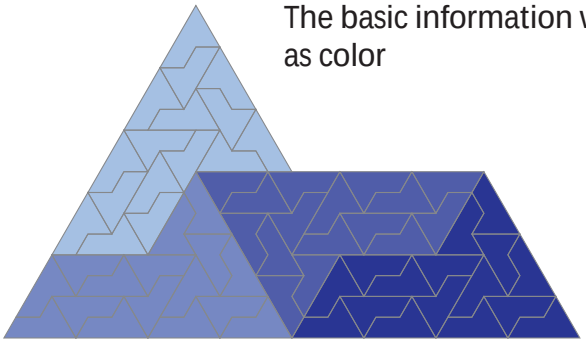
The (*) appears generally necessary because of subtleties "infinite" fault lines— what we actually prove is that in every tiling by the new tiles, the every tile lies in an infinite hierarchy of larger and larger supertiles.

The basic proof is standard for aperiodic heirarchical tilings— we induct, showing that the tiles clump into larger and larger versions of themselves.

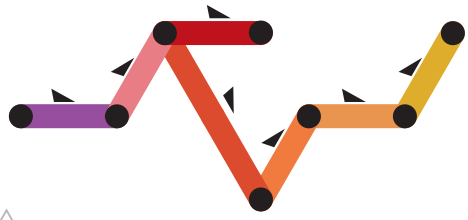
The main result, really, is a technique for achieving this in general.

The basic information about a supertile is its location within its parent

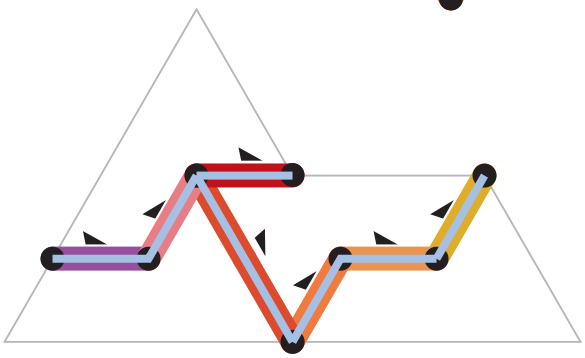
The basic information will be encoded as color

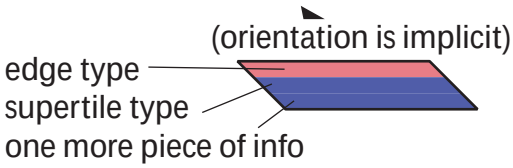


Information will be shuttled along a "skeleton" of edges. Note that every edge in a sphinx substitution tiling lies on the skeleton of a unique sized supertile. Edge edge then needs to know to what part of a skeleton it belongs: location, orientation and direction.



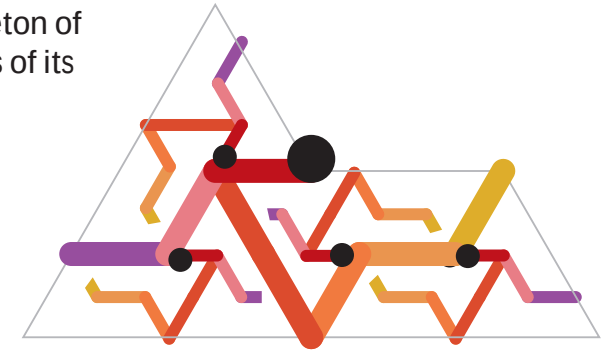
We can store information on the edge tiles

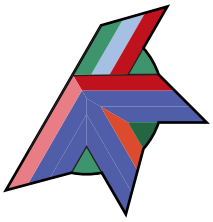




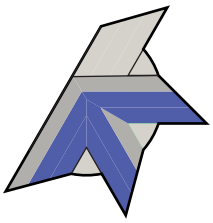
Vertices in the skeleton mediate information. For example, one kind of vertex tile solely handles this connection

Vertices also connect the skeleton of each supertile to the skeletons of its children and its parent

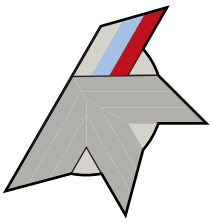




ensures skeleton is assembled correctly

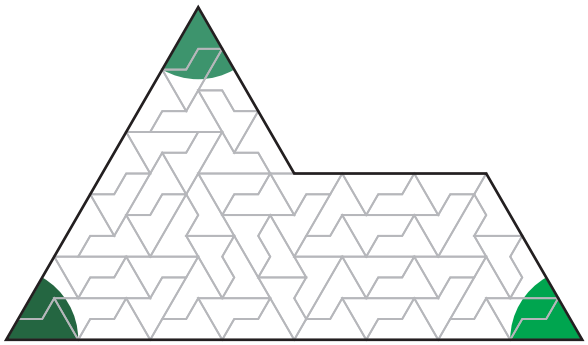


ensures skeleton has consistentent informtation

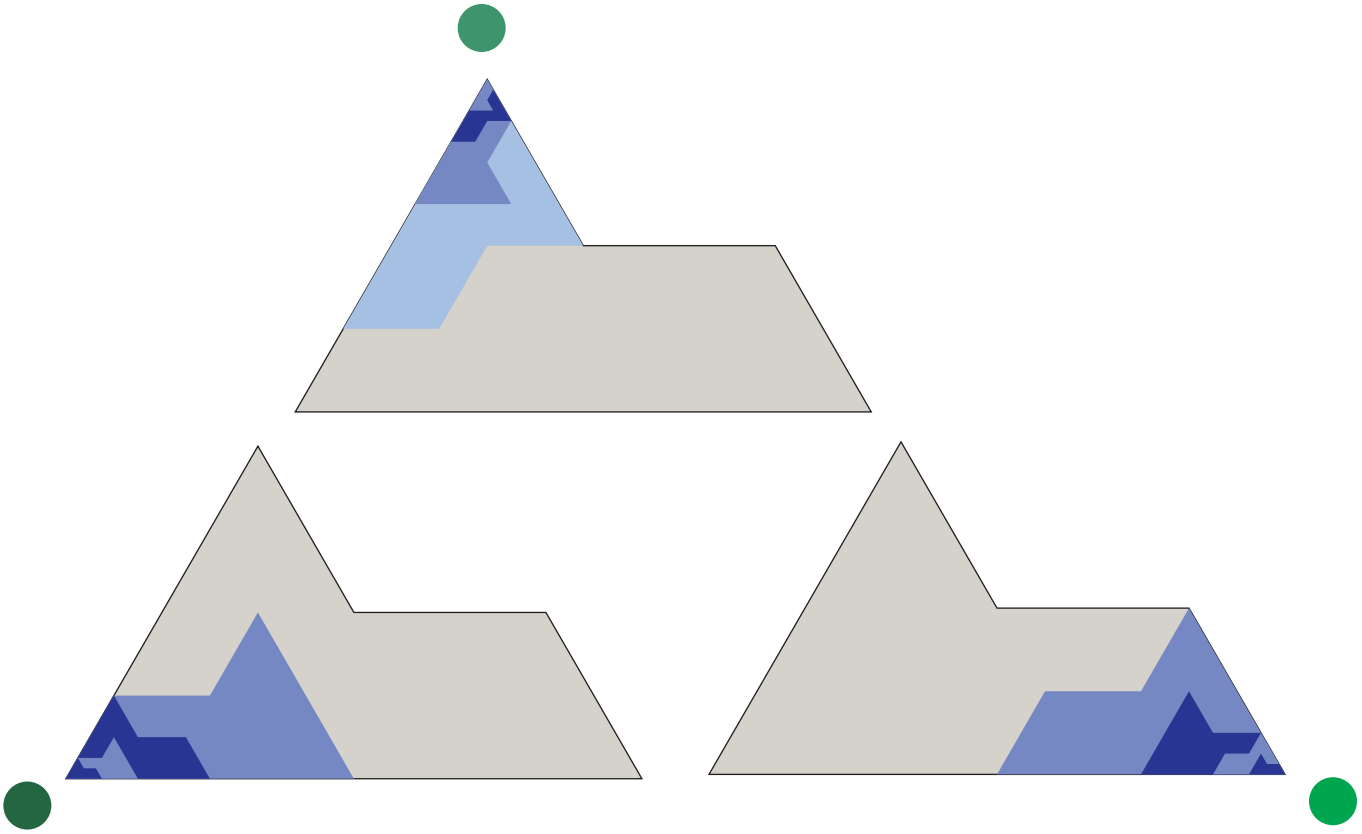


ensures correct child, meeting correctly, at this location

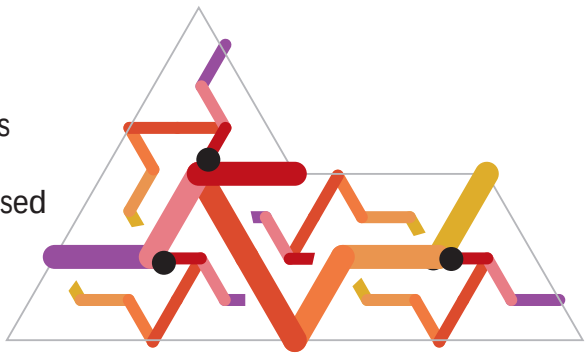
It is also helpful to know where the vertices of a supertile lie, especially where there are no internal edges. There are three such vertices in the sphinx. We mark these vertices by their location on the boundary of the largest possible supertile

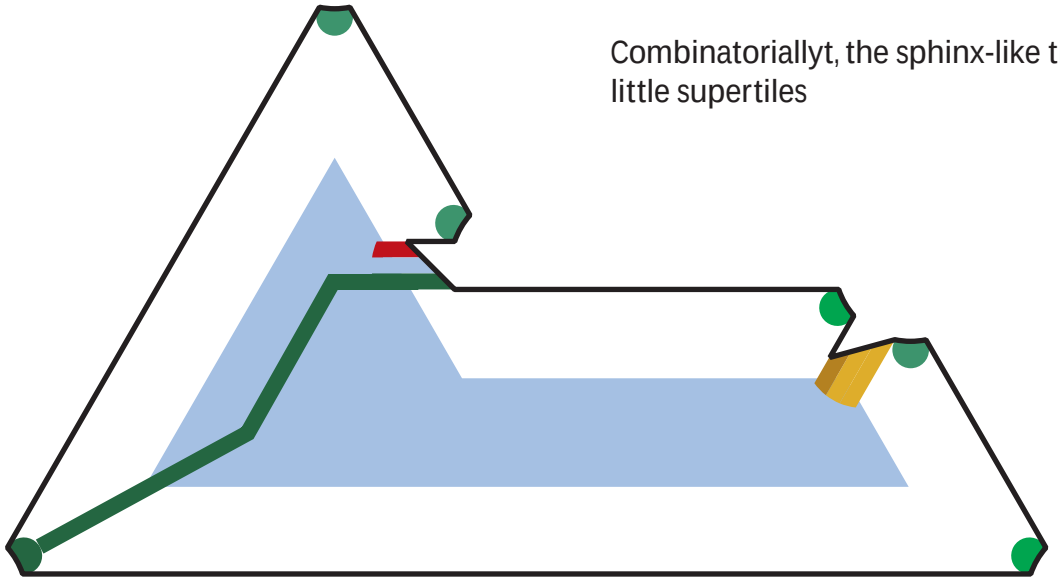


This information is shuttled down the hierarchy in the third channel along each edge:

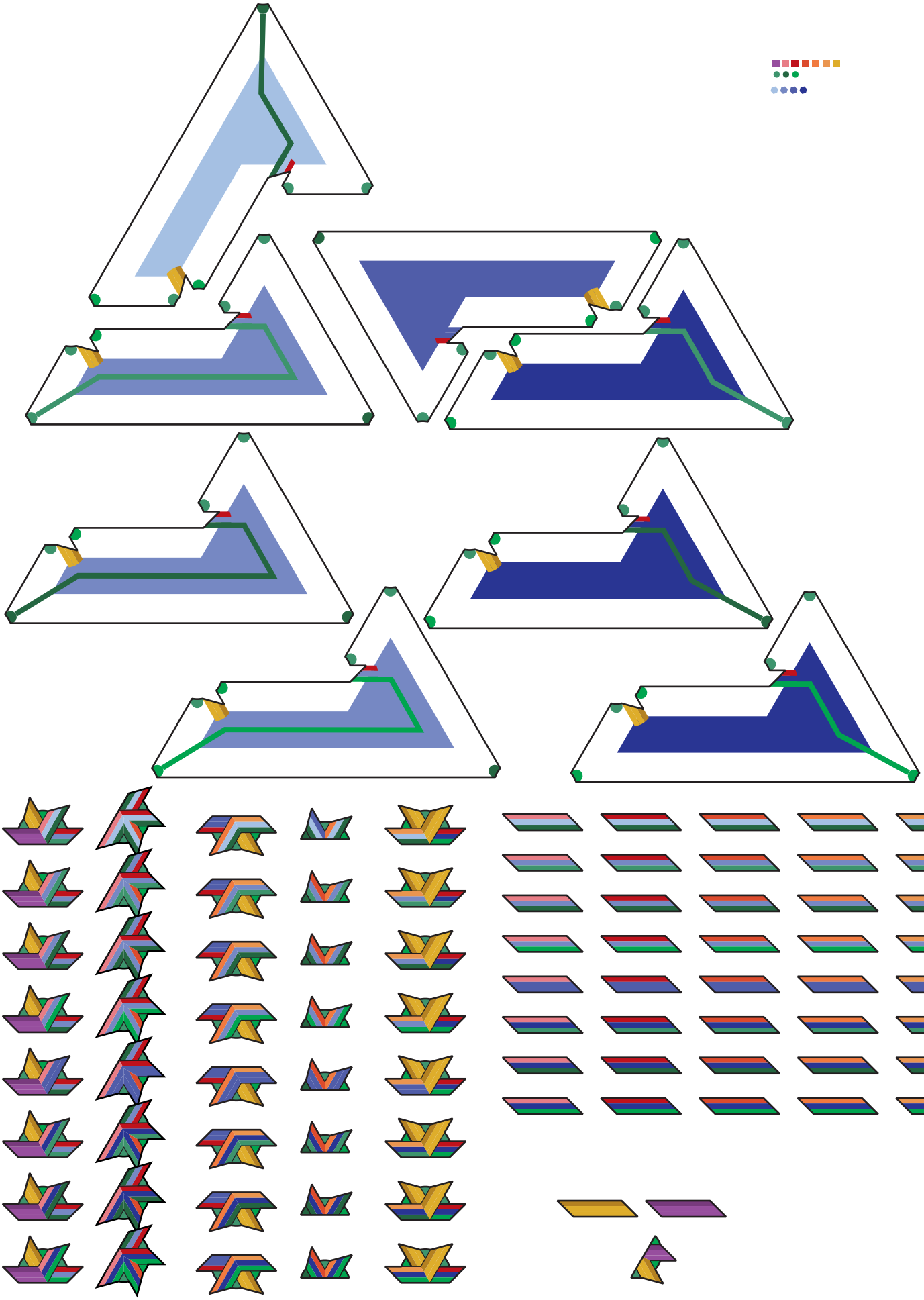


These three kinds of supertiles may have to shuttle this information; consequently, the vertices connection their skeletons to that of their parent has to ensure the correct information is being passed down.

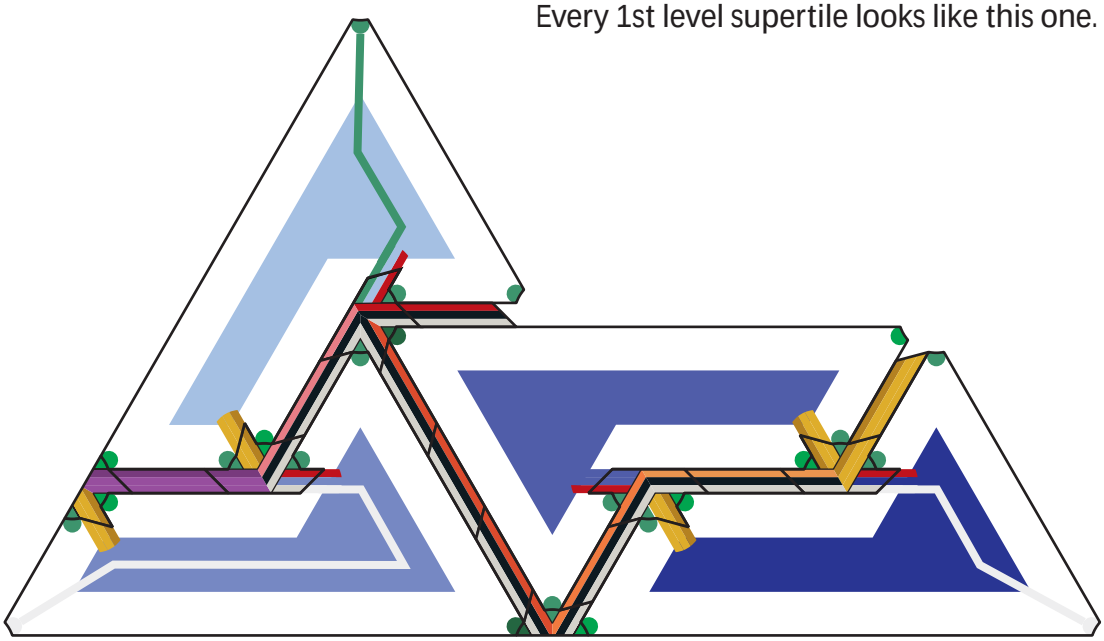




Combinatoriallyt, the sphinx-like tiles are just little supertiles



Every 1st level supertile looks like this one.



Every 2nd level supertile looks like this.