At the age of 24 Isaac Newton wrote his great Principia, the foundation of caculus and of modern physics.

In this book, he works out one physical principle after another, describing the motion of planets, the action of gravity, and any number of other fundamental aspects of the natural world, all by using a few basic tools.

In hindsight, it is clear how his invention of calculus shapes this work, but at the time, it could not have been obvious at all: Newton kept his discovery of calculus per se secret for another twenty years.

Principia is full, though, of the kinds of descriptions of motion we've been using here in class.

At each moment, an object has a position, and a velocity, and something (a force, typically) changing the velocity. In effect, this is all his three laws of motion state. Once we know the forces, we know how velocity changes, and then how the object moves around.

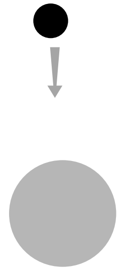
That's all well and good, but Newton has another powerful idea at his disposal; just like we have been doing, we can look at this step by step, like the frames of a movie.

If the frames come faster and faster, we more closely approximate the true motion of the object. Indeed, Newton felt comfortable using infinitely brief steps of time!

As we saw earlier, this very idea had caused some trouble since ancient times. Part of Newton's genius was to realize that the philosophical issues were irrelevant— whether or not infinitely short moments "exist", we may as well work as though they do.

(One hundred and fifty years later, Auguste Cauchy finally put this on firmer ground, as we have already discussed.)

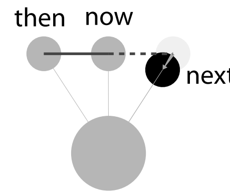
Let's see this in action!



Suppose a planet is near a star. The star's gravity pulls the planet towards it.

What this means is, no matter how the planet is moving, at the next step, its velocity direction will be pulled closer to the star.

For example:



As the planet moves around the star, it is constantly being pulled inwards.

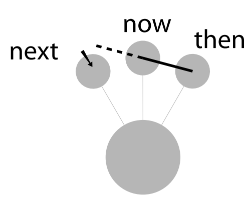
(One other thing: how strong is the force pulling on the planet? The further away, the less strong. The exact relationship is that if you double the distance, the force is one quarter the strength: that is, the force is proportional to the inverse squared distance.)

But why doesn't it just fall straight in? Basically, in a nutshell, it is rocketing past: it is always been pulled it, but its moving past the star at the same time.

If we take shorter and shorter time steps we get just the right answer: these kinds of paths describe ellipses. In fact, Kepler's three laws of planetary motion come from just this argument.

Special Case: To move in a circular path, velocity is always tangent to the circle, and the change in the velocity is just what we need to get back on track.

If we take infinitely short time steps, this exactly matches up; the forces all point right to the center of the circle



A falling ball.

Assume a falling ball has a starting value of dy to be 1, and a starting position of y to be 1.

At each step we bump up dy by 2, then bump up y by dy.

Try this out

step	dy	y
1	1	1
2	3	
3		
4		
5		

Another example:

The end of a spring is being pulled into place by a force exactly proportional how far out of whack it is.

In other words, if the end of the spring is x units out of its rest position, then dx changes by some multiple of x.

This is called Hooke's law.

What is calculus?

In all these problems we see changes, and accumulations of changes, and are asking about the relationship between them.

For example, if the end of a spring is moving according to Hooke's law, what kinds of paths can the end of a spring follow?

Somehow Hooke's law is relating the position of the spring to the change in the position, and calculus is a tool for working out that relationship.