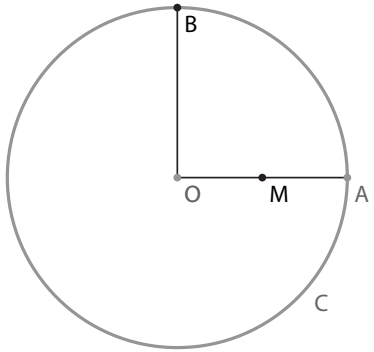
Four Constructions of a Regular Pentagon, Given a Circumscribing Circle.

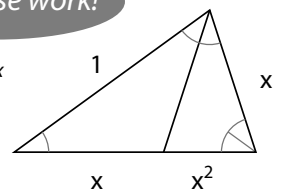
Math 3773 Fall 2010

All four constructions begin with a circle C with center O. Draw a radius OA and find its midpoint M. Draw a perpendicular radius OB.

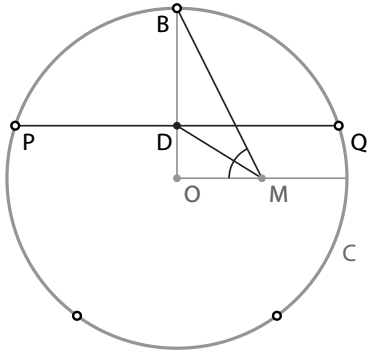


Prove these work!

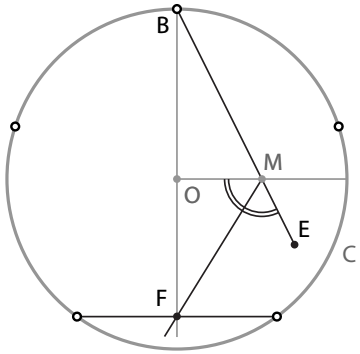
hint: solve for x
Show this x is constructed



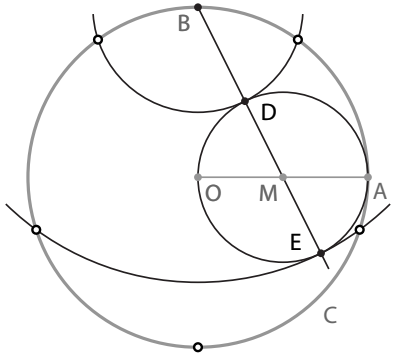
1. Richmond's construction: construct the bisector of angle OMB and let D be the intersection of OB with this bisector. Construct the line parallel to OM, passing through D and Let P and Q be the intersections of the circle C with this parallel. P, Q and B are three of the vertices of the desired pentagon.



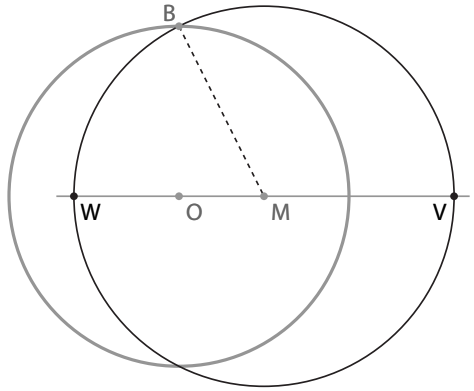
Two vertices are enough to find all the others, but it's nice to see how Richmond's construction can be continued: Extend the ray from B through M to some point E. As before, construct the bisector of angle OME, and let F be the intersection of this bisector with the ray BO. Constuct the parallel to OM passing through F—the other two vertices of the pentagon lie at the intersection of this parallel with C



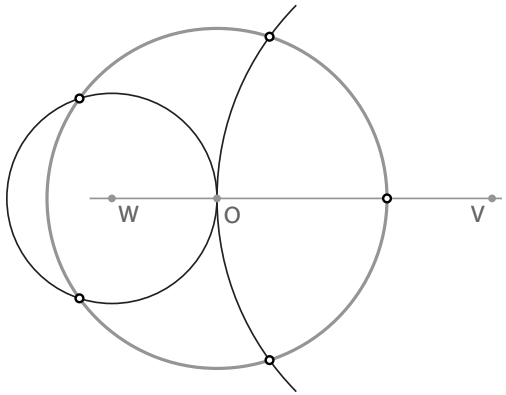
2. Draw the ray from B passing through M, and the circle with center M passing through O and A. Let D and E be the intersections of this ray with this circle. Draw the circle with center B passing through D, and the circle with center B passing through E. The intersections of these circles with the original circle C are four of the five vertices of the desired pentagon. The fifth vertex R is opposite B on the circumference of C.



For the third and fourth constructions, draw the circle with center M, passing through B, and let W and V be the intersections of this circle with the line OA.



3. Euclid's construction: Once we have V and W, construct the circles with centers V and W passing through O. The intersections of these circles with C are four of the five vertices of the desired pentagon. The fifth vertex is our our point A.



4. Or we can use "Carlyle circles": construct the circles with the same radius as the circle C, but centered at V and W. The intersections of these circles with C are four of the five vertices of the desired pentagon. The fifth is the point opposite A on the circumference of C.